FM Algebra Name: _____

Solving Absolute Value Equations Date: _____ Pd: _____

If $|x| = 4$, what values of *x* will make the sentence true?

Most students say *x* = 4. They are partially correct. The absolute value of 4 is 4. ($|4| = 4$) However, an equation is not solved unless all of the values that make a sentence true are found. Because the absolute value of -4 is also 4 ($|-4| = 4$), the solution must include both values. Therefore the solution is ***x* = 4, -4**.

If $|x + 3| = 5$, what values of *x* will make the sentence true?

Knowing that the absolute value of a positive or negative value will give us the same result, we need to see that the amount inside the absolute value can be either 5 or -5. This implies that two equations need to be true. They are that $x + 3 = 5$ or that $x + 3 = -5$. Solve both of these equations to find the solutions to the original absolute value equation.

$$\begin{aligned} x + 3 &= 5 \\ x &= 2 \end{aligned} \quad \text{or} \quad \begin{aligned} x + 3 &= -5 \\ x &= -8 \end{aligned}$$

Our solution is that ***x* = 2, -8**. We should check both solutions to see if they work.

Check:
$$|2 + 3| = 5 \qquad |-8 + 3| = 5$$
$$|5| = 5 \qquad |-5| = 5$$

Both of our solutions checked with the original equation.

Example: Solve $|y| = 7$.

Solution: ***y* = 7, -7**

Example: Solve $|m - 5| = 9$.

Solution: There are two implied equations to solve.

$$\begin{aligned} m - 5 &= 9 \\ m &= 14 \end{aligned} \quad \text{or} \quad \begin{aligned} m - 5 &= -9 \\ m &= -4 \end{aligned}$$

***m* = 14, -4**

Make sure you always check both solutions with the original equation.

Although you can expect most absolute value equations to have two solutions, there are two exceptions.

Example: Solve $|h| = 0$.

Solution: There is only one value that will give you an absolute value of zero.
***h* = 0**

Example: Solve $|p| = -2$.

Solution: It is impossible to find the absolute value of a number and get a negative value. Therefore there is no solution to an equation like this. Your response should be **No Solution**.

As always, our equations can get much more complicated. However, the following general principles should apply.

1. Isolate the absolute value on one side of the equation.
2. Set up two equations to solve for the expected two solutions.

Example: Solve $|x+3|+14 = 20$.

Step 1: Isolate the absolute value.

$$|x+3|+14 = 20$$
$$|x+3| = 6$$

Step 2: Set up the two equations.

$$x+3 = 6 \quad \text{or} \quad x+3 = -6$$
$$x = 3 \qquad\qquad x = -9$$

Solution: ***x* = 3, -9** Check both solutions with the original equation.

Example: Solve $2|3x+1|+1 = 5$.

Step 1: Isolate the absolute value.

$$2|3x+1|+1 = 5$$
$$2|3x+1| = 4$$
$$|3x+1| = 2$$

Step 2: Set up the two equations.

$$3x+1 = 2 \quad \text{or} \quad 3x+1 = -2$$
$$3x = 1 \qquad\qquad 3x = -3$$
$$x = \frac{1}{3} \qquad\qquad x = -1$$

Solution: $x = \frac{1}{3}, -1$ Check both solutions with the original equation.

Additional examples can be found on p. 390 of your Algebra textbook.

Solve each absolute value equation below. Show all steps and circle your answer. Make sure you check both of your solutions with the original equations.

1. $|x+4|=3$

2. $|6-x|=9$

3. $|x+12|=8$

4. $|2x+6|=14$

5. $|x-10|+4=14$

6. $|2x-4|+6=9$

7. $6+|-x-5|=9$

8. $|2x+9|-15=36$

9. $2|x+1|-1=9$

10. $3|x-7|+5=17$

11. $10+|3x+1|=24$

12. $|2-5x|-8=5$

13. $2|3x-7|+2=4$

14. $3|4x+11|-2=7$

15. $2|x+3|-3=-3$

16. $3|x-5|+15=12$